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Comparison of Real and Envelope Cross-Correlation Techniques

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INTRODUCTION

This document presents a summary and comparison of two different cross-correlation techniques: real correlation and envelope correlation by using the analytic signal. Data recorded in analog form are subsequently sampled and digitized for processing in a digital computer. The correlator operates on any real or complex signals and outputs the magnitude, phase, real part, and imaginary parts of the complex output. The digitized data can be directly input as real data for real correlation, or they can be input in the form of the complex-valued analytic signal for an envelope correlation.

REAL CORRELATION

The data are input into the correlator with only real parts, as shown in figure 1. The correlator assumes the imaginary part to be zero. The fast Fourier transform (FFT) of each channel is performed and the resulting complex spectra are Hermitian (real part exhibiting even symmetry; imaginary part exhibiting odd symmetry). One channel's spectrum then undergoes a complex conjugation. Since only the imaginary parts change their sign, the result is still Hermitian. Then the two spectra undergo a complex multiplication and the product is again Hermitian. Finally, the inverse fast Fourier transform (IFFT) of a Hermitian signal returns a real-valued sequence of data. Since the output is completely real, there are no phase angles other than 0 or $\pm\pi$ corresponding to positive and negative values of the correlation coefficient.

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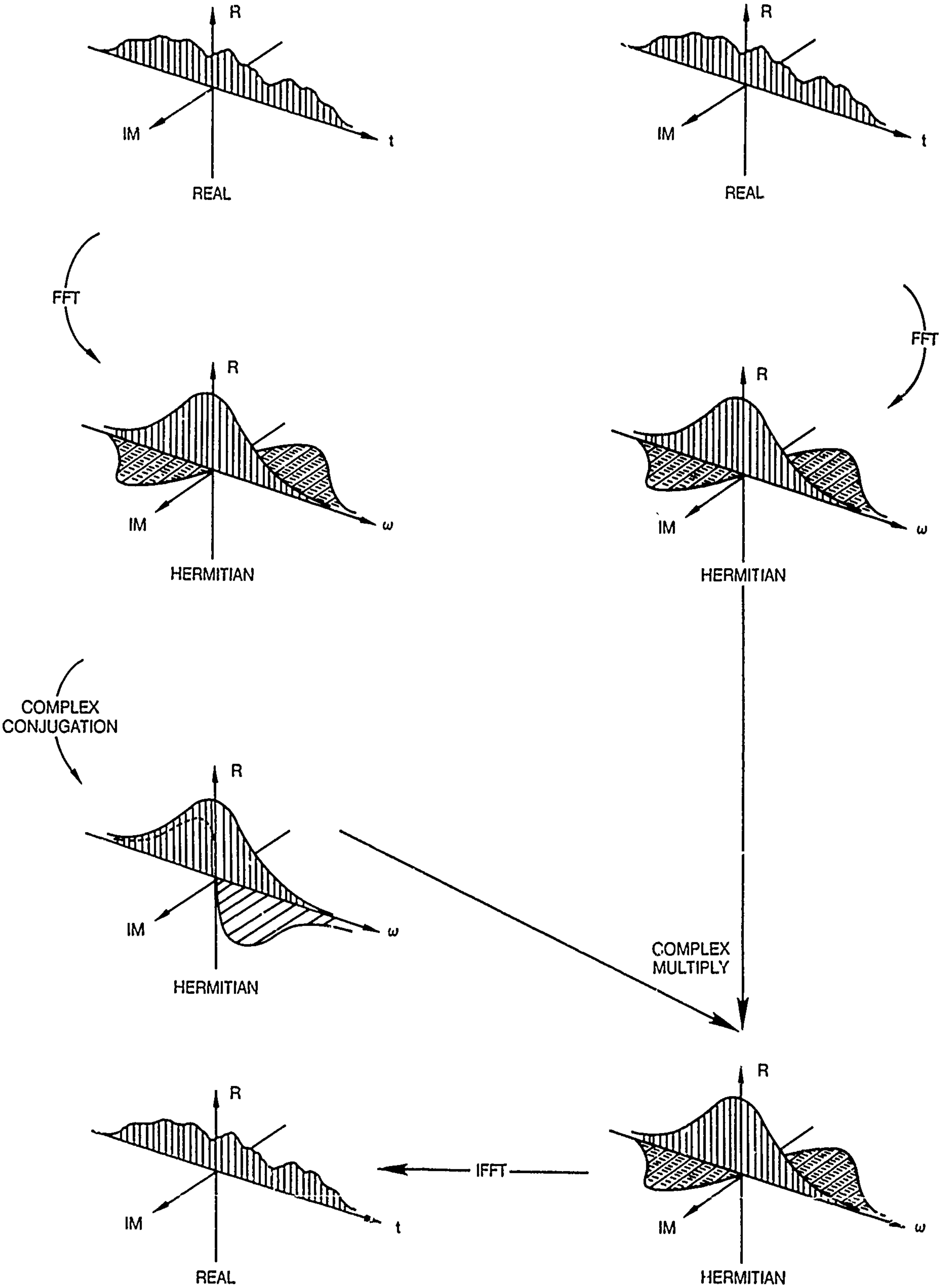


Figure 1. Real correlation.

ENVELOPE CORRELATION

Before the data are sent to the correlator, they are made into an analytic signal. The analytic signal $x_a(t)$ is a complex-valued signal formed from the original real signal $x(t)$, as shown in figure 2. The negative frequency values of the spectrum $X(f)$ are set to zero and the positive values are doubled. We have

$$x(t) \rightarrow X(f)$$

$$x_a(t) \rightarrow 2 \cdot u(f) \cdot X(f)$$

where $u(f)$ is the frequency domain unit step function. Multiplication in the frequency domain results in convolution in the time domain as

$$\left[\delta(t) - \frac{1}{j\pi t}\right] \leftarrow 2 \cdot u(f)$$

$$x(t) \leftarrow X(f)$$

$$x_a = x(t) \otimes \left[\delta(t) - \frac{1}{j\pi t}\right]$$

$$= x(t) + j \cdot \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau}\, d\tau$$

$$= x(t) + j \cdot \hat{x}(t)$$

where the real part of the analytic signal is just $x(t)$, and the imaginary part $\hat{x}(t)$ is the Hilbert transform of $x(t)$. The Hilbert transform pair is given as

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau}\, d\tau$$

$$x(t) = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\hat{x}(\tau)}{t - \tau}\, d\tau \ .$$

The analytic signal then can be summarized as:

$$\Re(x_a) = x(t)\ , \qquad \Im(x_a) = \hat{x}(t)\ , \qquad |x_a(t)| = \sqrt{x^2(t) + \hat{x}^2(t)}\ , \qquad \Phi(t) = \tan^{-1}\left(\frac{\hat{x}(t)}{x(t)}\right).$$

After each input channel has been converted to this analytic signal representation, they are sent through the correlator, as shown in figure 3. The complex-valued spectra exhibit no symmetry properties, because the negative region of frequency is zero-valued. As a result, the inverse FFT of the complex-conjugate multiplication is complex-valued. The resulting correlation has phase that may range from $-\pi$ to $+\pi$. The magnitude of this signal

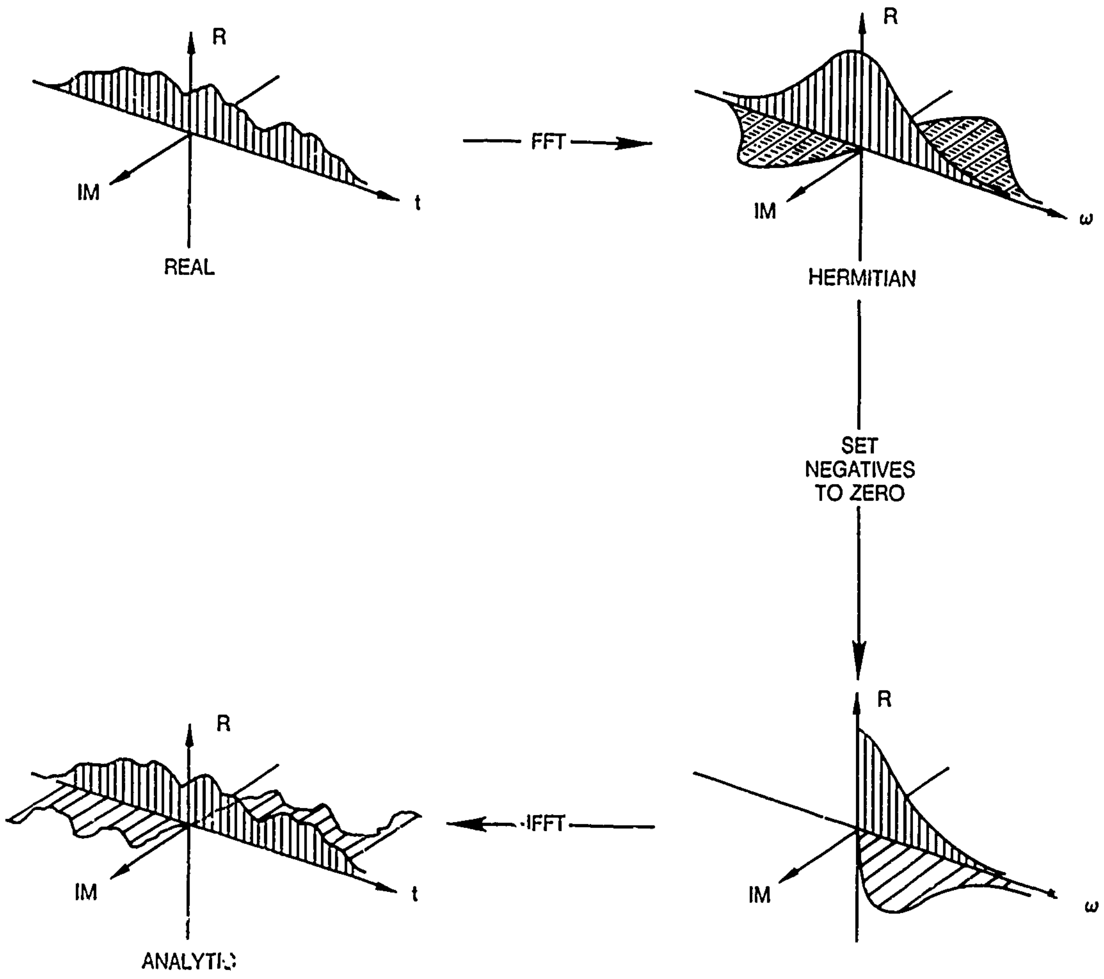


Figure 2. Formation of the analytic signal.

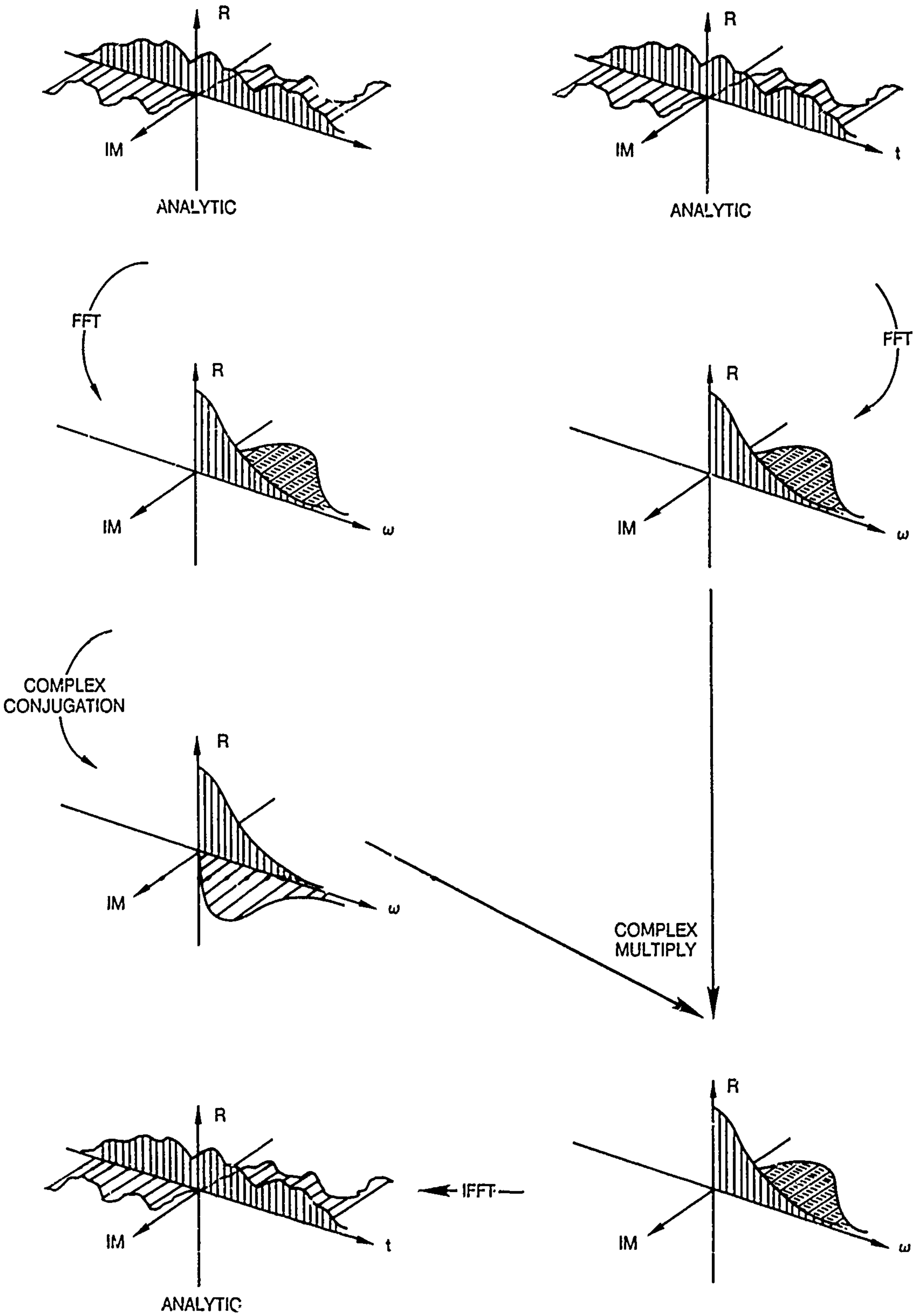


Figure 3. Envelope correlation.

is known as the envelope correlation. It is also interesting to note that the real part of it is equivalent to the output of the real correlation method. Therefore, it is possible to obtain both the real correlation and the envelope correlation by means of this method.

COMPARISONS

It is instructive to further examine the details of the the two correlation approaches. To do this, we consider a flat rectangular cross spectrum under the following conditions:

Case 1. Correlation of nonanalytic signals

- Full band, both positive and negative frequencies
- Bandpass, both positive and negative frequencies.

Case 2. Correlation of analytic signals.

- Full band, positive frequencies only
- Bandpass, positive frequencies only.

The first case corresponds to real correlation, the second to envelope correlation. We will show the modulating effects that bandpassed signals yield. We will also show that we can obtain the real correlation by the methods used in case 2.

CASE 1: REAL CORRELATION (NONANALYTIC SIGNALS)

In general, the FFT of the rectangular pulse is a sinc function, as shown in figures 4a and 4b.

$$A \cdot \text{rect}\left(\frac{f}{W}\right) \leftrightarrow AW \cdot \text{sinc}\,(Wt) = \frac{AW \cdot \sin\,(\pi Wt)}{\pi Wt} \qquad \text{for full band.}$$

We get a pure sinc function that is real-valued. This is the output we expect where the spectrum is flat across the whole band. It is also what we expect for basebanded bandpass signal, because the spectra look essentially the same. We now examine the effects of bandpassing the signal (figures 4c and 4d).

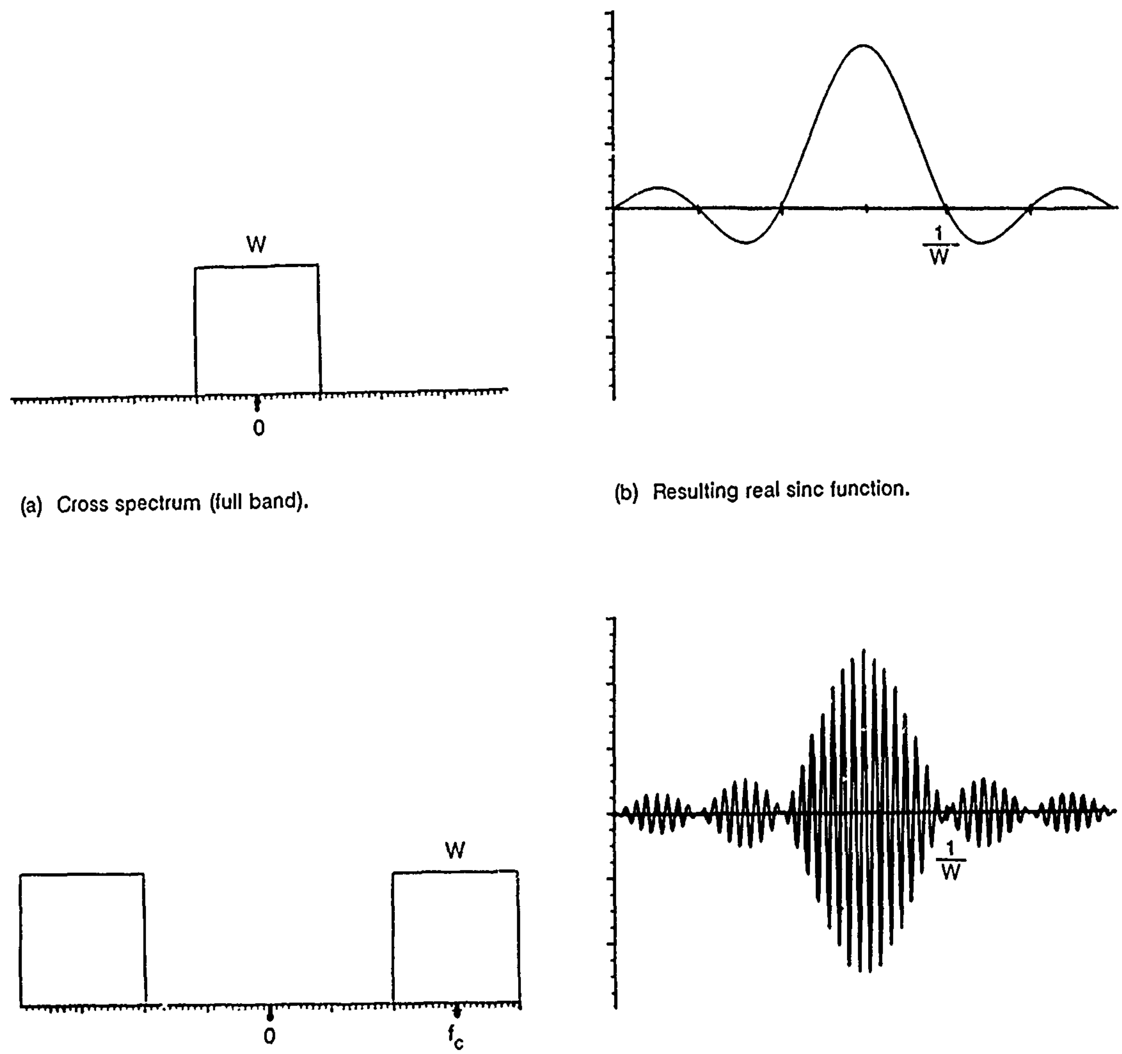
$$G(f) = A \cdot \text{Rect}\left(\frac{f - f_c}{W}\right) + A \cdot \text{Rect}\left(\frac{f - f_c}{W}\right)$$

$$g(t) = AW \cdot \exp\,(j2\pi f_c t) \cdot \text{sinc}\,(Wt) + AW \cdot \exp\,(-j2\pi f_c t) \cdot \text{sinc}\,(Wt)$$

$$= AW \cdot \text{sinc}\,(Wt)[\exp\,(j2\pi f_c t) + \exp\,(-j2\pi f_c t)]$$

$$= 2AW \cdot \sin\,(Wt) \cdot \cos\,(2\pi f_c t)$$

$$= \frac{2AW \cdot \sin\,(\pi Wt)}{\pi Wt} \cdot \cos\,(2\pi f_c t) \qquad \text{for bandpass.}$$



(a) Cross spectrum (full band).

(b) Resulting real sinc function.

(c) Cross spectrum (bandpass).

(d) Resulting modulated sinc function.

Figure 4. Real correlation (nonanalytic signals).

This result is also real-valued, but it is a sinc function that is modulated by a cosine of frequecy f_c. We can see this modulation and the effect of varying W and f_c in figures 5a through e. Whenever $W = 2f_c$, the modulated sinc becomes a pure sinc with twice the frequency of the envelope, as shown in figure 5f.

CASE 2: ENVELOPE CORRELATION (ANALYTIC SIGNALS)

In general, for a flat rectangular band of width W and centered at f_c we have

$$G(f) = A \cdot \mathrm{Rect}\left(\frac{f - f_c}{W}\right)$$

$$g(t) = AW \cdot \exp(j2\pi f_c t) \cdot \mathrm{sinc}(Wt)$$

$$= AW \,\mathrm{sinc}(Wt)[\cos(2\pi f_c t) + j \cdot \sin(2\pi f_c t)]$$

$$= AW \cdot [\mathrm{sinc}(Wt) \cdot \cos(2\pi f_c t) + j \cdot \mathrm{sinc}(Wt) \cdot \sin(2\pi f_c t)] \; .$$

This result may be expressed in terms of magnitude, phase, real part, and imaginary part for both bandpass and full-band ($W = 2f_c$) signals.

$$|g(t)| = AW[\mathrm{sinc}^2(Wt) \cdot \cos^2(2\pi f_c t) + \mathrm{sinc}^2(Wt) \cdot \sin^2(2\pi f_c t)]^{1/2}$$

$$= AW \cdot \mathrm{sinc}(Wt) \qquad \text{for full band and bandpass.}$$

$$\Phi(t) = \tan^{-1}\left[\frac{\mathrm{sinc}(Wt) \cdot \sin(2\pi f_c t)}{\mathrm{sinc}(Wt) \cdot \cos(2\pi f_c t)}\right]$$

$$= \tan^{-1}[\tan(2\pi f_c t)] = 2\pi f_c t \qquad \text{for bandpass.}$$

$$= 2\pi \frac{W}{2} = \pi W t \qquad \text{for full band.}$$

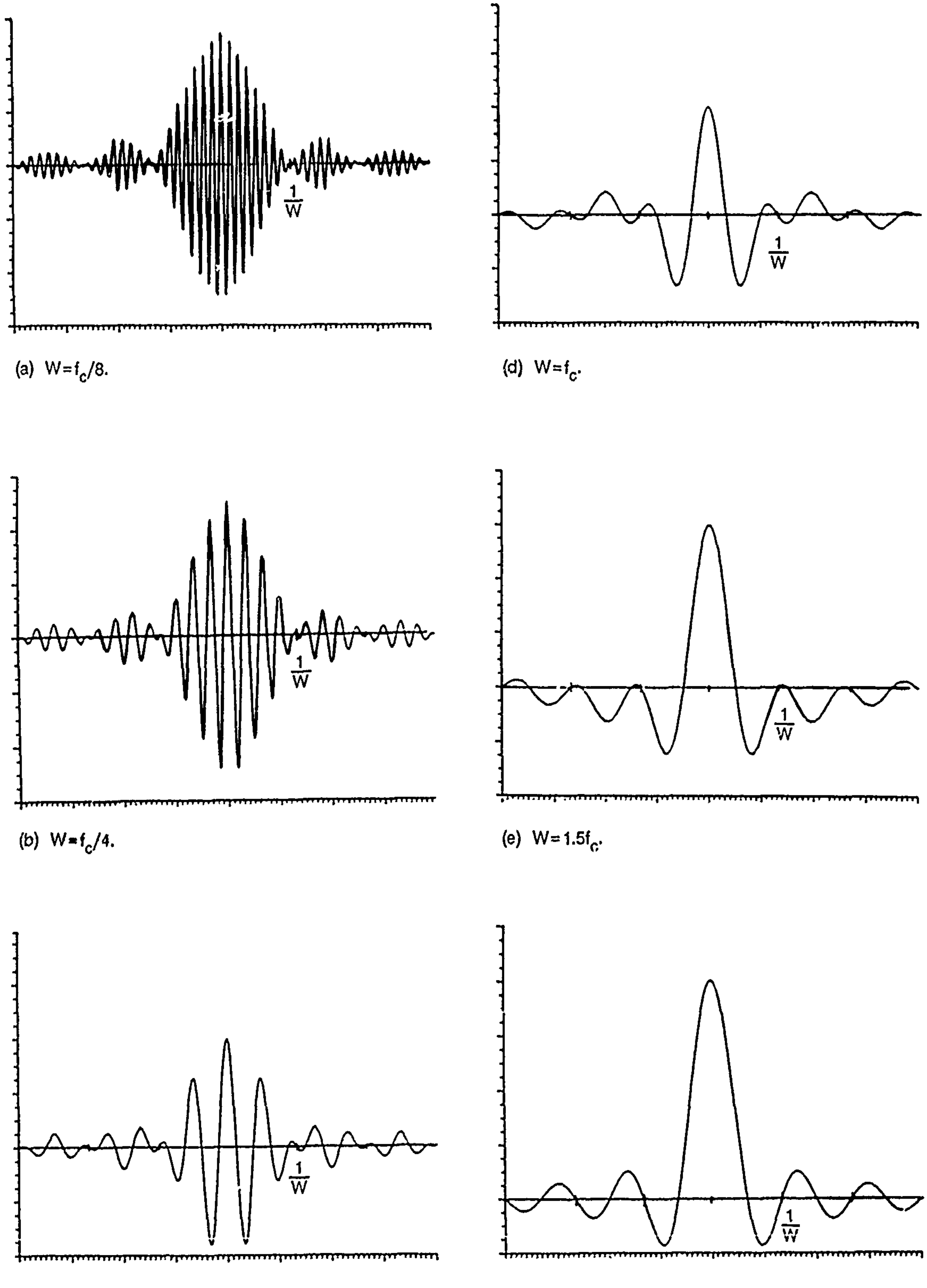
$$\Re\{g(t)\} = AW \cdot \mathrm{sinc}(Wt) \cdot \cos(2\pi f_c t) \qquad \text{for bandpass.}$$

$$= \frac{AW \sin(2\pi Wt)}{2\pi Wt} = AW \,\mathrm{sinc}(2Wt) \qquad \text{for full band.}$$

$$\Im\{g(t)\} = AW \cdot \mathrm{sinc}(Wt) \cdot \sin(2\pi f_c t) \qquad \text{for bandpass.}$$

$$= AW \cdot \mathrm{sinc}(Wt) \cdot \sin(\pi Wt) \qquad \text{for full band.}$$

These full-band and bandpass correlations are shown, respectively, in figures 6 and 7. In either case, the magnitudes are equivalent and are known as the envelope correlation. It is a sinc function dependent only on the bandwidth W. The phase of the correlation shows rotations through 2π at the frequency center (f_c) of the band. The real and imaginary parts are sinc functions modulated by sinusoidal carriers of frequency f_c. The real parts are modulated by a cosine, and the imaginary parts by a sine. Since we are dealing here with analytic signals, the imaginary part is just the Hilbert transform of the real part. As we observed earlier, in the case where $W = 2f_c$ (full band or basebanded bandpass), the modulated sinc becomes a pure sinc of twice the envelope frequency. It is also interesting to observe that the real parts of the envelope correlations (case 2) are equivalent to the corresponding real correlations (case 1). Therefore, we may obtain the real correlations' results by



(a) $W = f_c/8$.

(b) $W = f_c/4$.

(c) $W = f_c/2$.

(d) $W = f_c$.

(e) $W = 1.5 f_c$.

(f) $W = 2 f_c$.

Figure 5. Modulated sinc function.

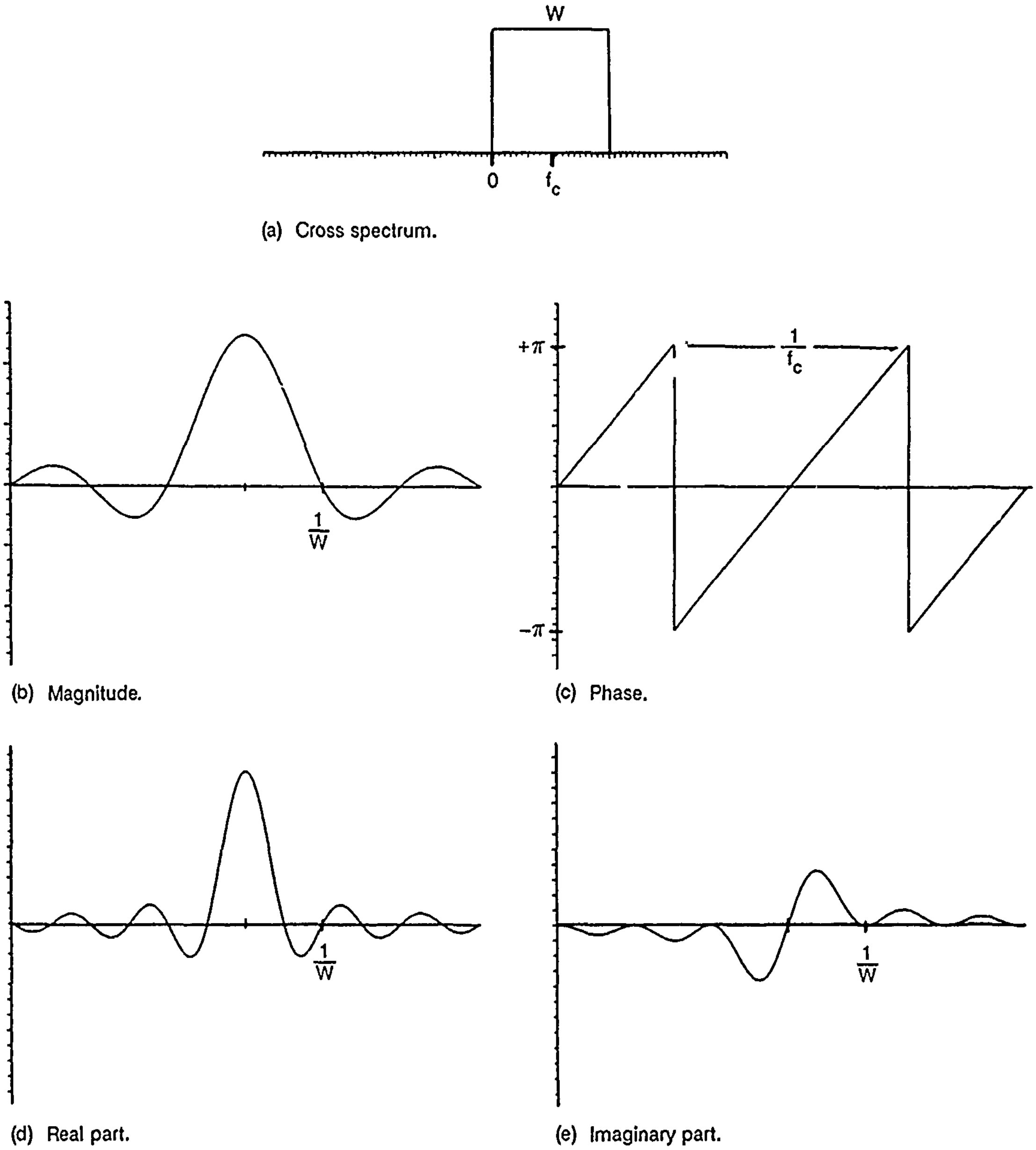


Figure 6. Full-band envelope correlation (analytic signals).

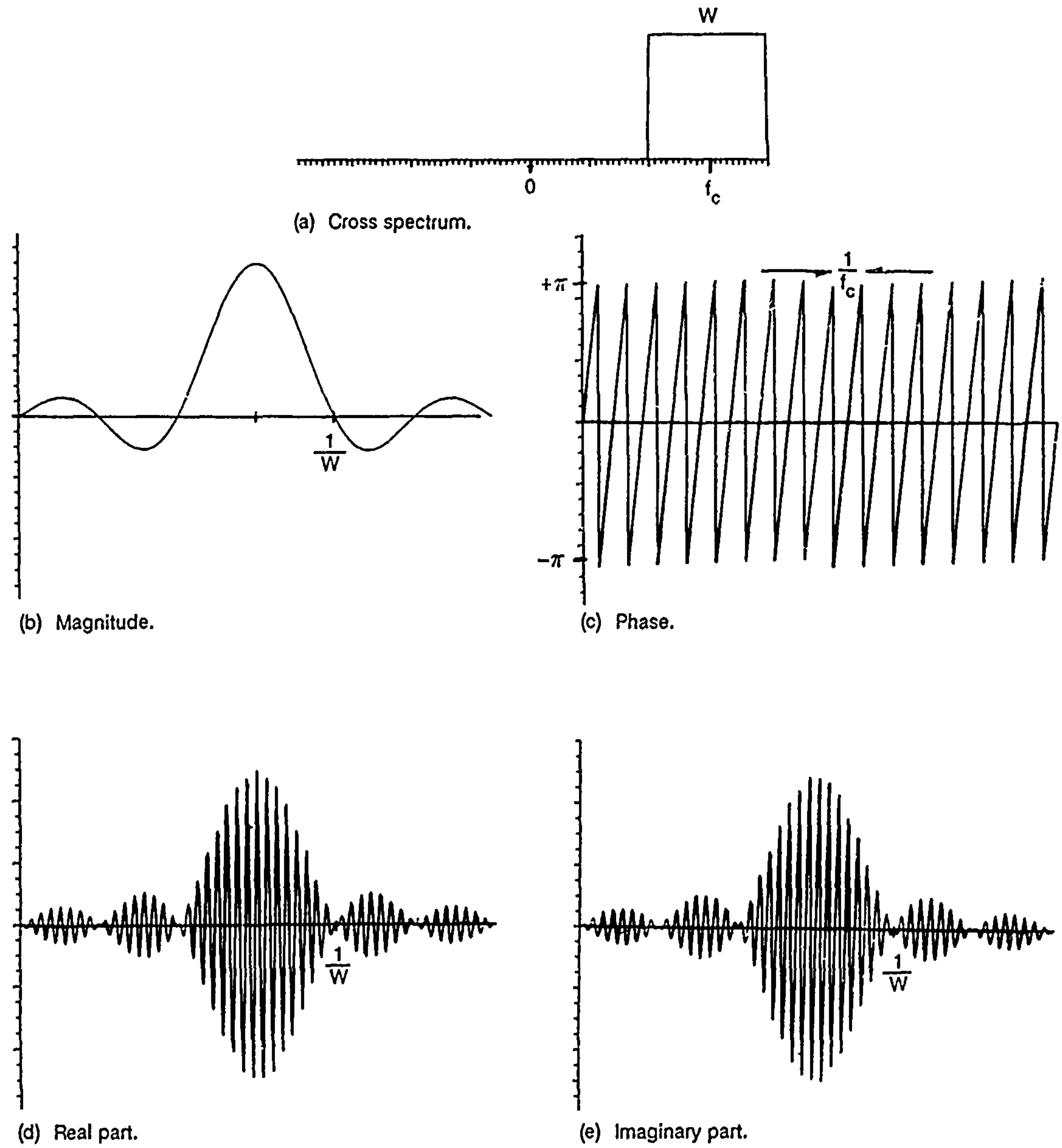


Figure 7. Bandpass envelope correlation (analytic signals).

means of the envelope correlation, by looking at the real part. The envelope method of processing is advantageous in that it gives a more complete and thorough representation of the correlation function.

EXAMPLES

We now look at the correlation method, using actual data and processing software. First, we attempt to reproduce the results obtained previously for a flat power spectrum. This is approximated by applying software (deline and delta mean equalizer) to flatten the power spectrum. Then the data are bandpass filtered with bandwidth $W = 200$ Hz centered at $f_c = 400$ Hz. In this manner, we approximate a flat rectangular band for both full band (baseband) and bandpass, as shown in figure 8. The resulting envelope correlations' magnitudes and phases are shown in figure 9. We can also see the real and imaginary parts in figure 10. The correlation results are close to and consistent with what we expect. Figure 11 shows the envelope correlations in a correlogram display. For this type of display, the minimum value is assigned a white pixel, and the maximum value a black pixel, setting the limits of the gray scale. The magnitude correlograms will always range from zero to 1, and the phase from $-\pi$ to $+\pi$. The real and imaginary correlograms range from the minimum correlation (negative values) to the maximum correlation values.

Next, we look at the cross correlation between signal recorded at two physically separated locations. We view them in terms of the correlogram outputs. Figure 12 shows the correlations for the full band, performed by both the real and the envelope methods. We observe the magnitude, phase, real, and imaginary parts of the correlation. Again, we see that the real part of the envelope correlation is equivalent to the real correlation. The imaginary part of the real correlation is zero, with any minute deviations spread across the entire gray scale. Figure 13 shows the effects of bandpassing the signal. We see both the envelope and the modulated real correlation. Finally, in figure 14 we see the effect of basebanding the input data.

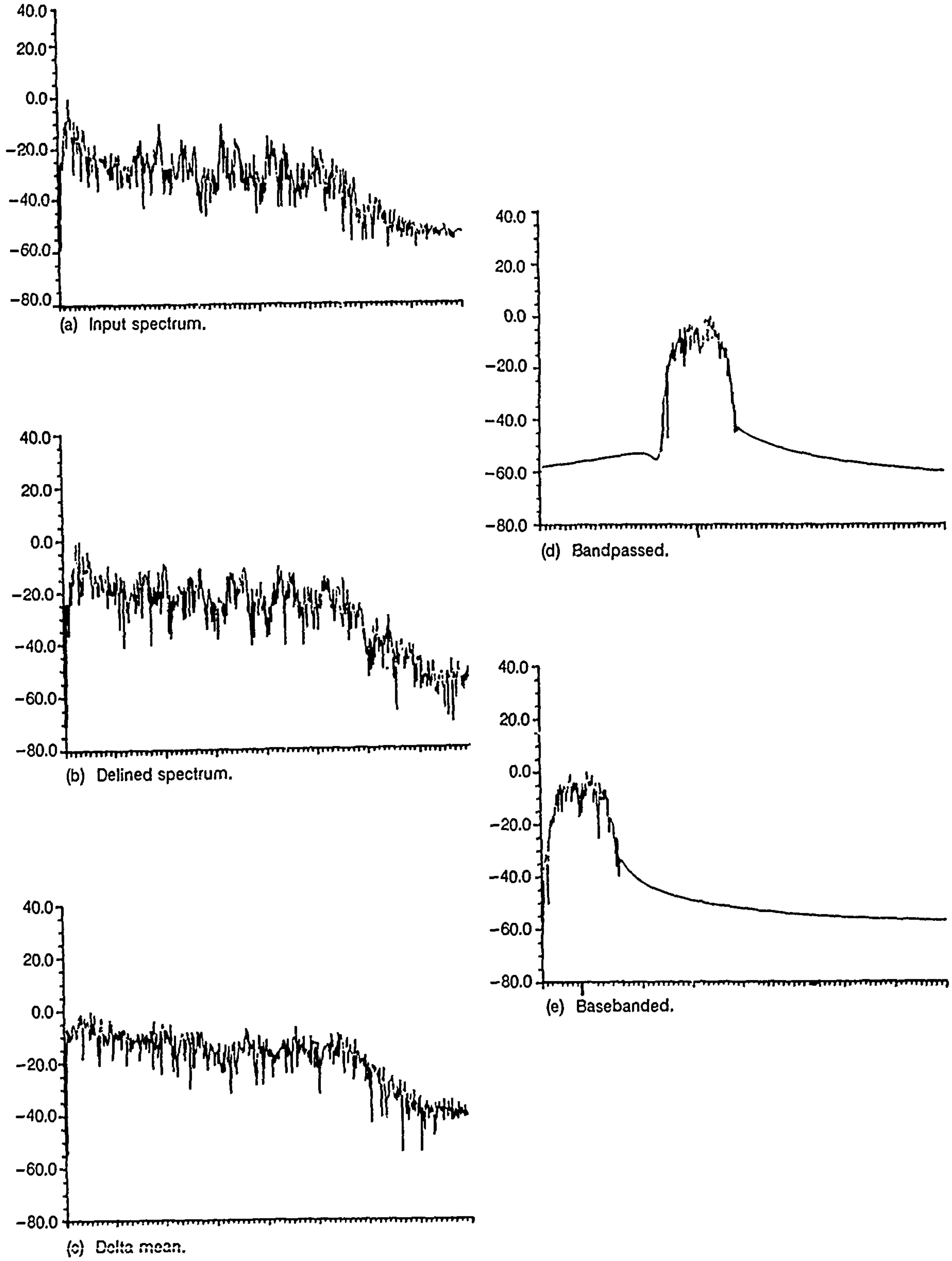
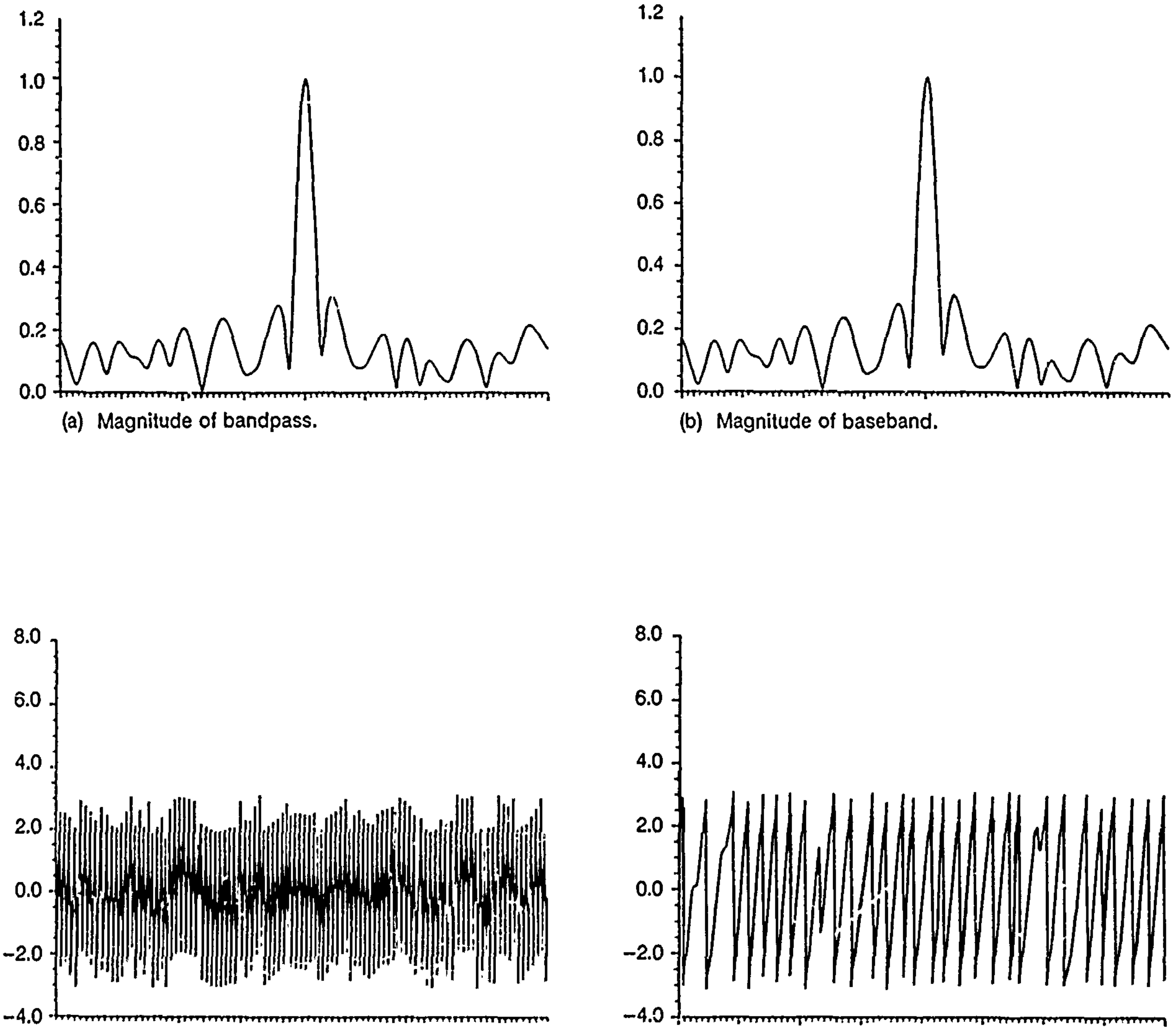


Figure 8. Approximation of flat rectangular spectrum, using actual data.



(a) Magnitude of bandpass.

(b) Magnitude of baseband.

(c) Phase of bandpass.

(d) Phase of baseband.

Figure 9. Autocorrelation of figure 8 (1).

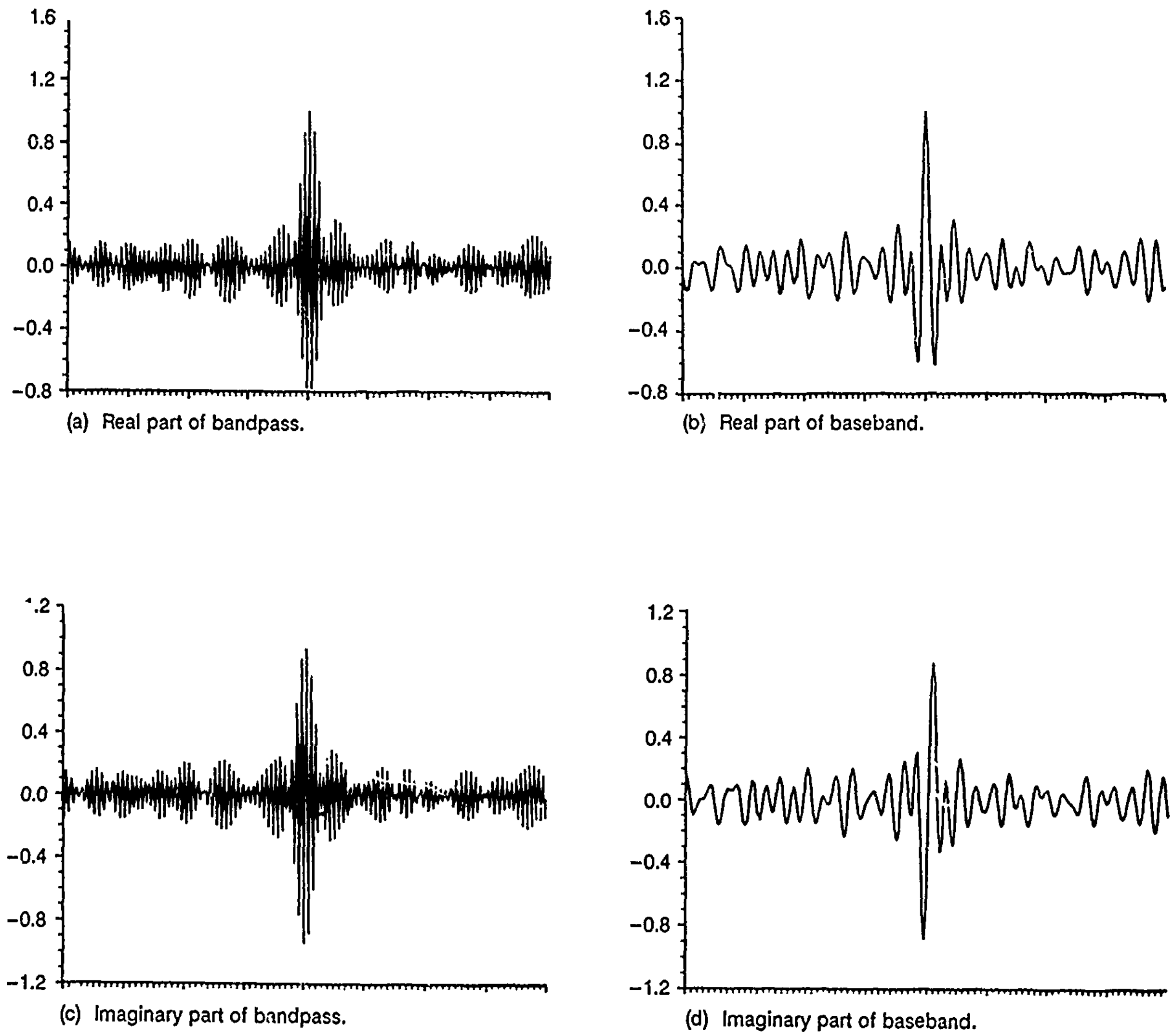


Figure 10. Autocorrelation of figure 8 (2).

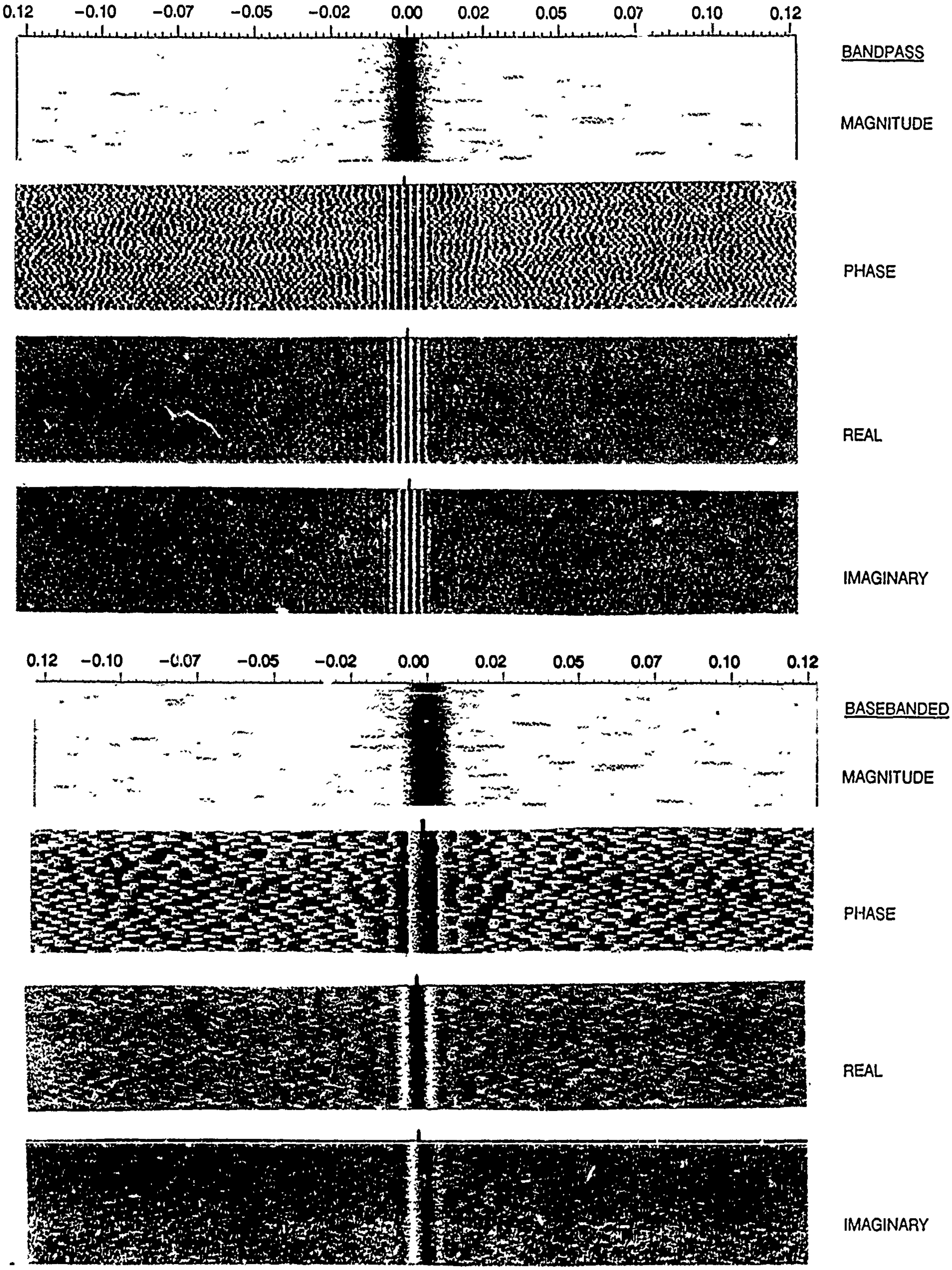


Figure 11. Envelope correlograms of figure 8 (bandpas, and basebanded).

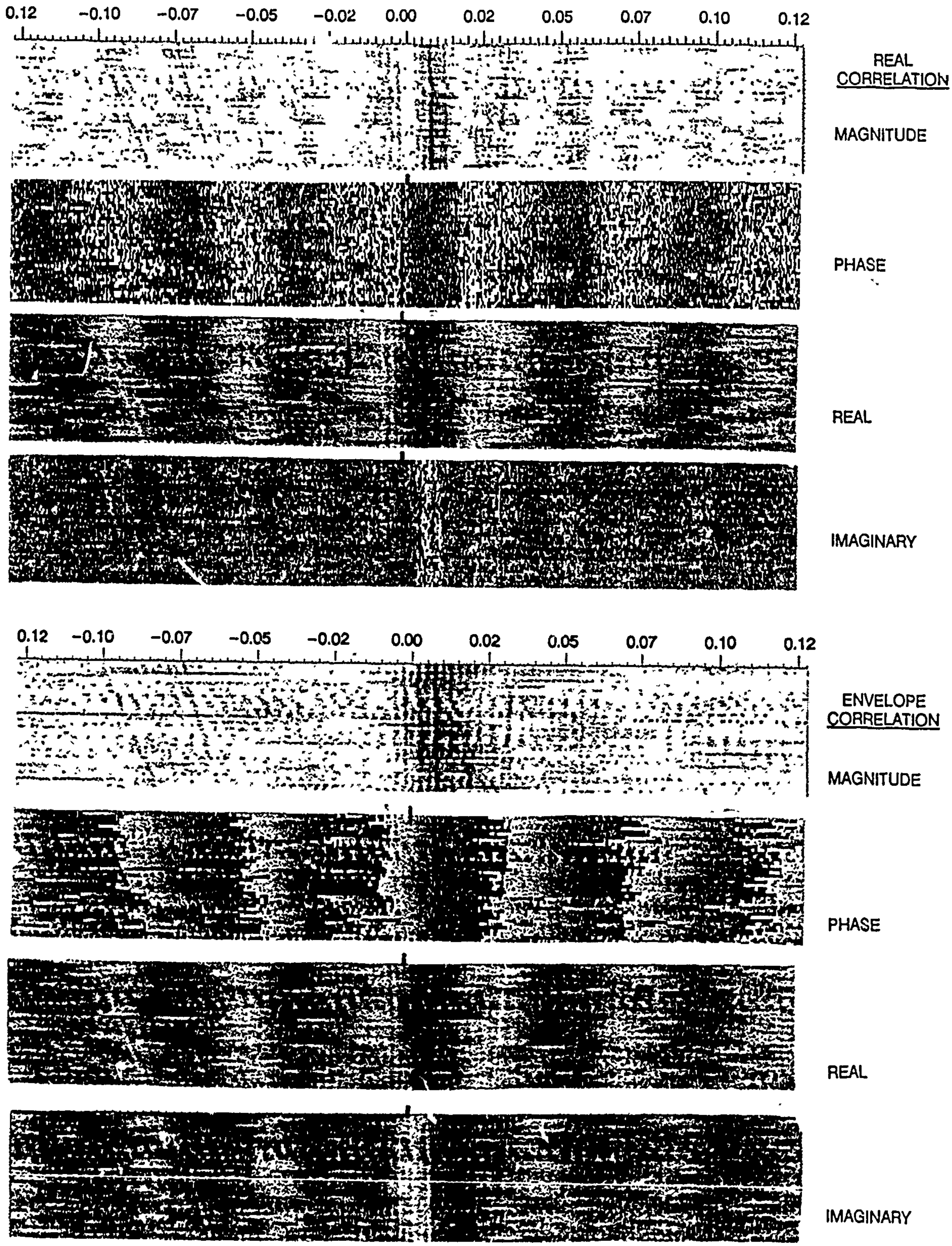


Figure 12. Sensor data correlograms, full band (real and envelope).

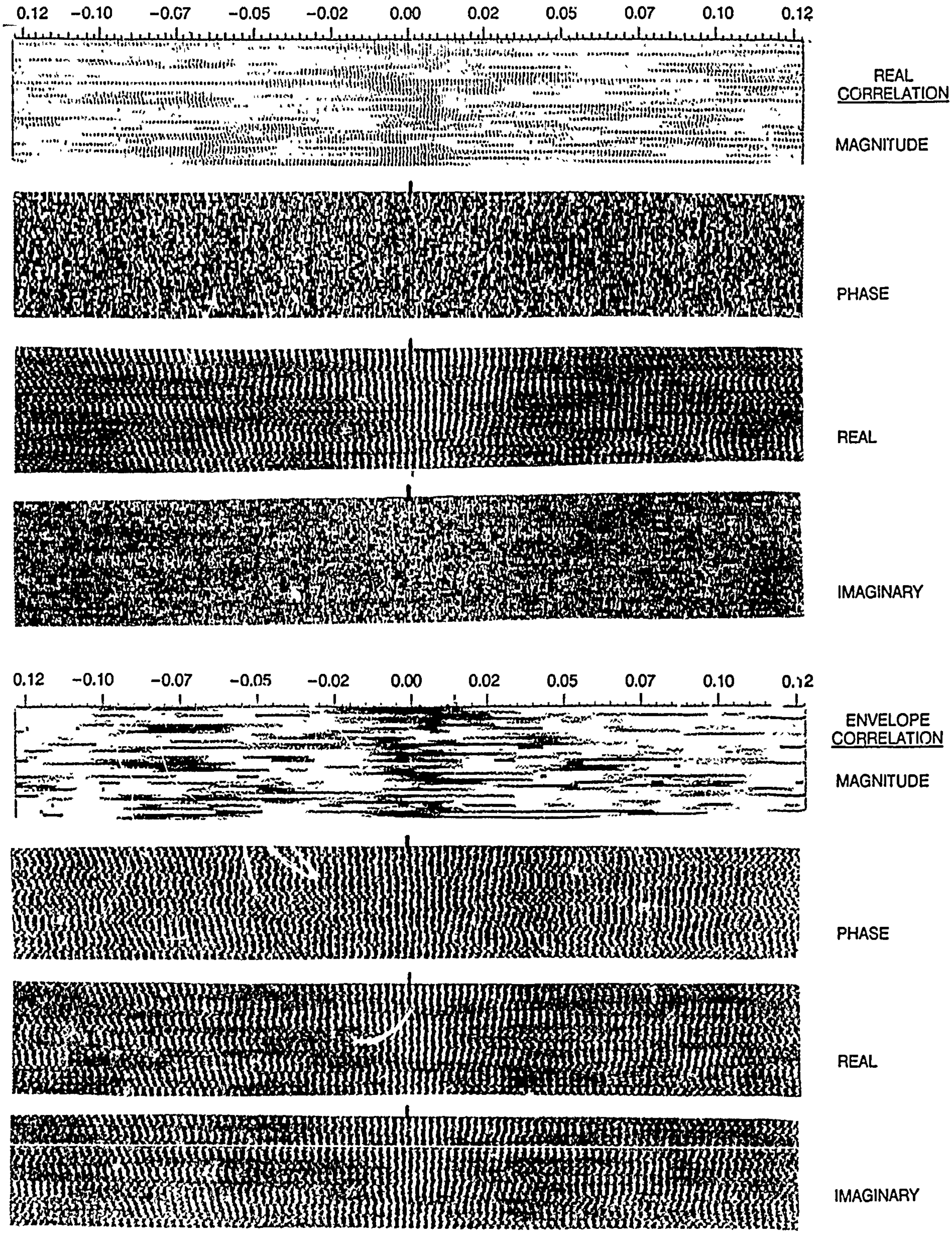


Figure 13. Sensor data correlograms, bandpass (real and envelope).

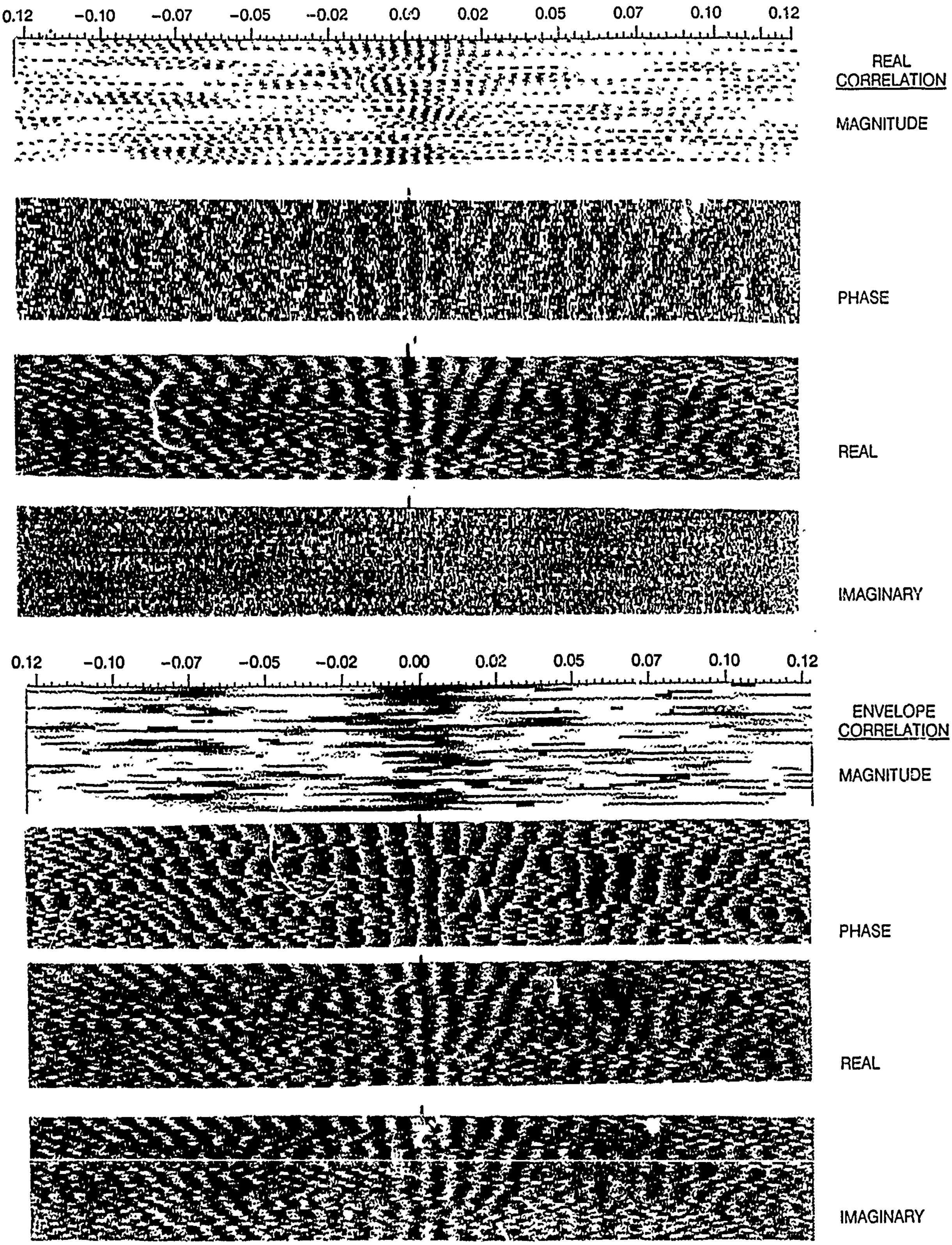


Figure 14. Sensor data correlograms, basebanded (real and envelope).

SUMMARY

We have described two methods of processing, real and envelope, that can be used to correlate signals. It is possible to obtain the real correlation with the envelope correlation method. It is advantageous to process with the analytic signal because it provides both of these correlation functions. With both correlation functions available, one may then choose which is better suited for the job.

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A summary and comparison of two different cross-correlation techniques are presented: real correlation and envelope correlation. Digitized data can be directly input as real-valued data for real correlation, or they can be input in the form of the complex-valued analytic signal for envelope correlation. Both a graphical and a mathematical description of each method are presented. Also shown is how the real correlation output can be obtained by the envelope correlation method.

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